1. Ski Jump

1.1 Definition of functions and derivatives

```
> restart;
  # set up curve and points
  y := a*x^4+b*x^3+c*x^2+d*x + e;
  dy := diff(y,x);
```

$$y := a\,x^4 + b\,x^3 + c\,x^2 + d\,x + e$$
$$dy := 4\,a\,x^3 + 3\,b\,x^2 + 2\,c\,x + d$$

1.2 Definition of points and tangencies

```
> P1 := [0,80];
  P2 := [120,0];
  P3 := [140,5];
  T1 := [0, 0];
  T2 := [120, 0];
```

$$P1 := [0, 80]$$
$$P2 := [120, 0]$$
$$P3 := [140, 5]$$
$$T1 := [0, 0]$$
$$T2 := [120, 0]$$

1.3 Substitution to obtain coefficients

```
> eq1 :=  subs(x=P1[1],y) = P1[2];
  eq2 :=  subs(x=P2[1],y) = P2[2];
  eq3 :=  subs(x=P3[1],y) = P3[2];
  eq4 :=  subs(x=T1[1],dy) = T1[2];
  eq5 :=  subs(x=T2[1],dy) = T2[2];
```

$$eq1 := e = 80$$
$$eq2 := 207360000\,a + 1728000\,b + 14400\,c + 120\,d + e = 0$$
$$eq3 := 384160000\,a + 2744000\,b + 19600\,c + 140\,d + e = 5$$
$$eq4 := d = 0$$
$$eq5 := 6912000\,a + 43200\,b + 240\,c + d = 0$$

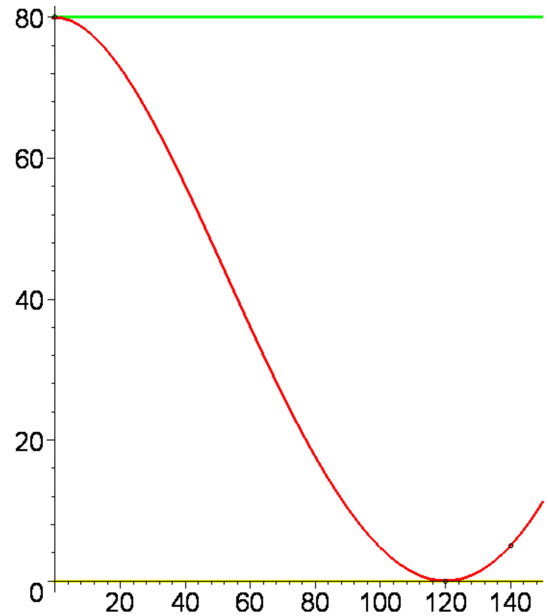
1.4 Solve equations to obtain function

```
> csol := solve({eq1,eq2,eq3, eq4, eq5},{a,b,c,d,e});
> ys := subs(csol,y);
  t1 := P1[2] + T1[2]*(x-P1[1]);
  t2 := P2[2] + T2[2]*(x-P3[1]);
```

$$csol := \{d = 0,\ a = \frac{-13}{42336000},\ b = \frac{11}{66150},\ c = \frac{-31}{1470},\ e = 80\}$$
$$ys := -\frac{13}{42336000}\,x^4 + \frac{11}{66150}\,x^3 - \frac{31}{1470}\,x^2 + 80$$
$$t1 := 80$$
$$t2 := 0$$

1.5 Graph

```
> with(plots):
  display({plot([ys, t1, t2],x= -1..150, thickness=3),plot([P1,P2,P3], color=black, style=point, symbol=circle)});
Warning, the name changecoords has been redefined
```



2. Cocktail Glass

2.1 Definition of functions and constants

```
> restart;
  # set up curve and points
  y := a*x^6+b*x^5+c*x^4+d*x^3+e*x^2+f*x+g;
  dy := diff(y,x);
  d2y := diff(y,x,x);
  r := 0.375;
```

$$y := a\,x^6 + b\,x^5 + c\,x^4 + d\,x^3 + e\,x^2 + f\,x + g$$
$$dy := 6\,a\,x^5 + 5\,b\,x^4 + 4\,c\,x^3 + 3\,d\,x^2 + 2\,e\,x + f$$
$$d2y := 30\,a\,x^4 + 20\,b\,x^3 + 12\,c\,x^2 + 6\,d\,x + 2\,e$$
$$r := 0.375$$

2.2 Definition of points and curvature

```
> P1 := [-1.5,3.5];
  P2 := [0, 0];
  P3 := [1.5,3.5];
  T1 := [-1.5, tan(-Pi/6)];
  T2 := [1.5, tan(Pi/6)];
  T3 := [0, 0];
  C1 := [0,2.6667];
```

$$P1 := [-1.5, 3.5]$$
$$P2 := [0, 0]$$
$$P3 := [1.5, 3.5]$$
$$T1 := \left[-1.5, -\frac{\sqrt{3}}{3}\right]$$
$$T2 := \left[1.5, \frac{\sqrt{3}}{3}\right]$$
$$T3 := [0, 0]$$
$$C1 := [0, 2.6667]$$

2.3 Substitution for coefficients

```
> eq1 :=  subs(x=P1[1],y) = P1[2];
  eq2 :=  subs(x=P2[1],y) = P2[2];
  eq3 :=  subs(x=P3[1],y) = P3[2];
  eq4 :=  subs(x=T1[1],dy) = T1[2];
  eq5 :=  subs(x=T2[1],dy) = T2[2];
  eq6 :=  subs(x=T3[1],dy) = T3[2];
  eq7 :=  subs(x=C1[1],d2y) = C1[2];
```

$$eq1 := 11.390625\,a - 7.59375\,b + 5.0625\,c - 3.375\,d + 2.25\,e - 1.5\,f + g = 3.5$$
$$eq2 := g = 0$$
$$eq3 := 11.390625\,a + 7.59375\,b + 5.0625\,c + 3.375\,d + 2.25\,e + 1.5\,f + g = 3.5$$
$$eq4 := -45.56250\,a + 25.3125\,b - 13.500\,c + 6.75\,d - 3.0\,e + f = -\frac{\sqrt{3}}{3}$$

$$eq5 := 45.56250\,a + 25.3125\,b + 13.500\,c + 6.75\,d + 3.0\,e + f = \frac{\sqrt{3}}{3}$$

$$eq6 := f = 0$$

$$eq7 := 2\,e = 2.6667$$

2.4 Solving for equation of curve and olive/cherry

```
> csol := solve({eq1,eq2,eq3,eq4,eq5,eq6,eq7},{a,b,c,d,e,f,g});
> ys := subs(csol,y);
  r := 0.375;
  bcirc := r-sqrt(r^2-x^2);
  tcirc := r+sqrt(r^2-x^2);
  t1 := P1[2] + T1[2]*(x-P1[1]);
  t2 := P3[2] + T2[2]*(x-P3[1]);
```

$$csol := \{ b = 0., c = 0.8033407009, e = 1.333350000, a = -0.3131478561, g = 0., f = 0., d = 0. \}$$

$$ys := -0.3131478561\,x^6 + 0.8033407009\,x^4 + 1.333350000\,x^2$$

$$r := 0.375$$

$$bcirc := 0.375 - \sqrt{0.140625 - x^2}$$

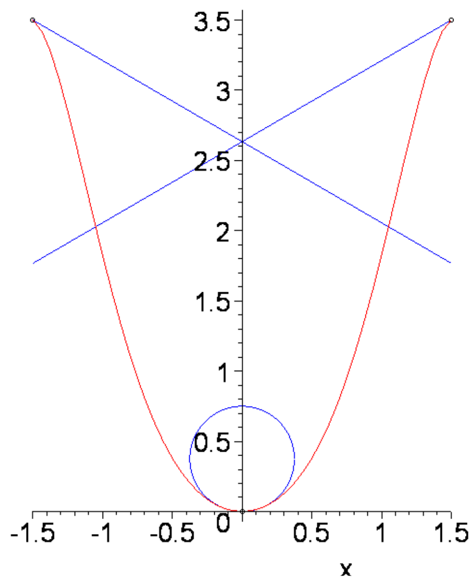
$$tcirc := 0.375 + \sqrt{0.140625 - x^2}$$

$$t1 := 3.5 - \frac{\sqrt{3}\,(x + 1.5)}{3}$$

$$t2 := 3.5 + \frac{\sqrt{3}\,(x - 1.5)}{3}$$

2.5 Graph

```
> with(plots):
  display({plot([ys,t1,t2],x= -1.5..1.5, color=[red,blue,blue]),
           plot([tcirc,bcirc], x= -r..r, color= blue),
           plot([P1,P2,P3], color=black, style=point, symbol=circle)},
           scaling = constrained );
Warning, the name changecoords has been redefined
```



2.6 Determination of Curvature

```
> semicircle := sqrt((x)^2 + (z - r)^2);
  simplify(diff(semicircle, x, x));
```

$$semicircle := \sqrt{x^2 + (z - 0.375)^2}$$

$$\frac{8.\,(64.\,z^2 - 48.\,z + 9.)}{(64.\,x^2 + 64.\,z^2 - 48.\,z + 9.)^{(3/2)}}$$

```
>
```

3. Cubic Splines

3.1 Definition of functions

```
> restart;
  # set up curve and points
  p := a*x^3+b*x^2+c*x+d;
  dp := diff(p,x);
```

$$p := a\,x^3 + b\,x^2 + c\,x + d$$

$$dp := 3\,a\,x^2 + 2\,b\,x + c$$

```
> q := e*x^3+f*x^2+g*x+h;
  dq := diff(q,x);
```

$$q := e\,x^3 + f\,x^2 + g\,x + h$$
$$dq := 3\,e\,x^2 + 2\,f\,x + g$$

3.2 Definition of points and tangencies

```
> P1p := [0,0];
  P2p := [1,1];
  T1p := [0, 0];
  T2p := [1, 0];
```

$$P1p := [0, 0]$$
$$P2p := [1, 1]$$
$$T1p := [0, 0]$$
$$T2p := [1, 0]$$

```
> P1q := [1,1];
  P2q := [2,0];
  T1q := [1,0];
  T2q := [2,0];
```

$$P1q := [1, 1]$$
$$P2q := [2, 0]$$
$$T1q := [1, 0]$$
$$T2q := [2, 0]$$

3.3 Substitution to obtain coefficients

```
> eq1 :=  subs(x=P1p[1],p) = P1p[2];
  eq2 :=  subs(x=P2p[1],p) = P2p[2];
  eq3 :=  subs(x=T1p[1],dp) = T1p[2];
  eq4 :=  subs(x=T2p[1],dp) = T2p[2];
```

$$eq1 := d = 0$$
$$eq2 := a + b + c + d = 1$$
$$eq3 := c = 0$$
$$eq4 := 3\,a + 2\,b + c = 0$$

```
> eq5 :=  subs(x=P1q[1],q) = P1q[2];
  eq6 :=  subs(x=P2q[1],q) = P2q[2];
  eq7 :=  subs(x=T1q[1],dq) = T1q[2];
  eq8 :=  subs(x=T2q[1],dq) = T2q[2];
```

$$eq5 := e + f + g + h = 1$$
$$eq6 := 8\,e + 4\,f + 2\,g + h = 0$$
$$eq7 := 3\,e + 2\,f + g = 0$$
$$eq8 := 12\,e + 4\,f + g = 0$$

3.4 Solve for equations

```
> csolp := solve({eq1,eq2,eq3,eq4},{a,b,c,d});
> ps := subs(csolp,p);
```

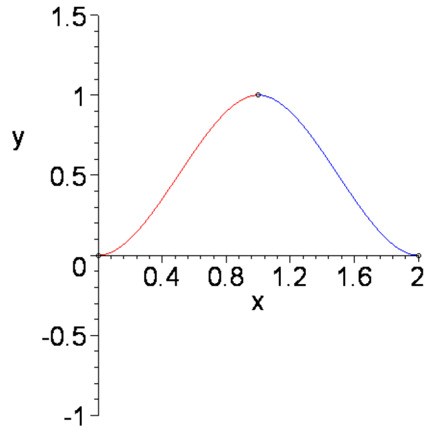
$$csolp := \{ a = -2, d = 0, c = 0, b = 3 \}$$
$$ps := -2\,x^3 + 3\,x^2$$

```
> csolq := solve({eq5,eq6,eq7,eq8},{e,f,g,h});
  qs := subs(csolq,q);
```

$$csolq := \{ g = 12, e = 2, f = -9, h = -4 \}$$
$$qs := 2\,x^3 - 9\,x^2 + 12\,x - 4$$

3.5 Graph

```
> with(plots):
  display({plot(ps,x= 0..1, y=-1..1.5),
           plot(qs,x= 1..2, y=-1..1.5, color=blue),
           plot([P1p,P2p,P2q], color=black, style=point, symbol=circle)},
           scaling = constrained );
Warning, the name changecoords has been redefined
```



1.5
1
0.5
0
-0.5
-1
y
0.4
0.8
1.2
1.6
2
x

>

>